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PARAMETER IDENTIFICATION AND OPTIMAL CONTROL OF GROUND TEMPERATURE

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SUMMARY

To avoid the use of pesticides on putting greens of golf courses, a temperature regulator system is strongly recommended nowadays in Japan. To maintain grass on the putting green without pesticide, the temperature of the ground should be controlled. This system consists of a cooling machine and buried pipes in the ground. The temperature of the water in the pipes cannot be regulated. In this paper, both identification and control problems are presented by the minimization technique and applied to a practical problem. To establish the system, it is important to obtain accurate parameters which are included in the governing equation. These parameters can be determined by parameter identification. The conjugate gradient method is used for the parameter identification procedure. The control problem aims to make the temperature at arbitrary points close to the objective temperature. The discrete-time dynamic programming is used for the control procedure.

KEY WORDS: identification; optimal control; finite element method; temperature control system

INTRODUCTION

Agricultural pesticides have long been used for farming purposes. Recently, these pesticides have also been applied to maintain recreational places, and have been especially heavily employed to maintain the grass on golf courses throughout the year. However nowadays, health hazards created by these toxic materials have aroused public concern about the life environment.

In Japan, many studies are conducted to control the usage of fertilizers and pesticides at least for recreational activities. A research institute in Chiba prefecture conducted an experimental study on the chemicals used in golf courses. They recommended ground level temperature control instead of pesticides for the cold region plants that are mainly used on the putting greens of golf courses to control the humidity and maintain a high temperature. According to that study, ground level temperature can be maintained by using water pipes under the ground. However, this water temperature could not be regulated adequately. In this study, it will be seen that temperature could be regulated by an optimal control theory. To express the heat conduction phenomenon, a two-dimensional heat conduction equation is used as the governing equation. The parameters that are unknown are included in the governing equation. If the values are not selected properly, any computed result will be meaningless. Thus, the parameters should be determined by the identification procedure.[1–3] Implementation of the parameter identification needs several observed data. These data are observed by the thermometers that are buried in the ground at the experimental field. An optimal control analysis is implemented by using these parameters. The temperature of water is determined by the criterion that minimizes the error between the temperature at an arbitrary point and the desired temperature.

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The purposes of this paper are to present methods for establishing the heat control system of the ground and to apply these methods to the practical model. The Fletcher–Reeves method used for the identification procedure and the discrete-time dynamic programming used for the control procedure are combined with the finite element method.

GOVERNING EQUATION

A two-dimensional heat conductivity equation is used to describe the behaviour of temperature in the analytical domain:

$$\rho C \frac{\partial T}{\partial t} - \beta\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \Omega \tag{1}$$

where β is the thermal conductivity, Ω is the internal heat generation rate and ρC is the volumetric heat capacity which is density times specific heat.

The boundary conditions may be specified by temperature or heat flux as

$$T = \hat{T} \quad \text{on } \Gamma_1 \tag{2}$$

$$\beta \frac{\partial T}{\partial n} = \hat{q} \quad \text{on } \Gamma_2 \tag{3}$$

where n is the outward unit normal to the boundary. The definition of boundary conditions and analytical domain are shown in Figure 1.

PARAMETER IDENTIFICATION

The finite element equation can be expressed in the following form by discretizing equation (1) in spatial variables:

$$\overline{\mathbf{M}(\gamma)}\,\dot{\mathbf{T}}(\boldsymbol{\beta}) + \mathbf{S}(\boldsymbol{\beta})\,\mathbf{T}(\boldsymbol{\beta}) = \Delta\hat{\boldsymbol{\Omega}} \tag{4}$$

where $\overline{\mathbf{M}(\gamma)}$, $\mathbf{S}(\boldsymbol{\beta})$ and $\Delta\hat{\boldsymbol{\Omega}}$ indicate a lumped mass matrix that is a linear function of γ, a diffusion coefficient matrix that is a linear function $\boldsymbol{\beta}$ and a thermal load vector that involves the boundary condition and heat generation, in which γ means ρC. The finite element equation is discretized in temporal variable by applying the explicit Euler method. The resulting equation is written as

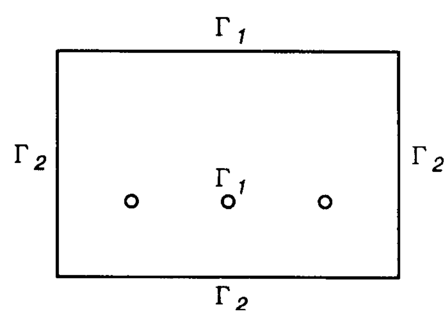

Figure. 1. Definition of boundary

follows:

$$\mathbf{T}^{n+1}(\boldsymbol{\beta}) = \mathbf{T}^{n}(\boldsymbol{\beta}) + \Delta t\overline{\mathbf{M}(\gamma)}^{-1}(-\mathbf{S}(\boldsymbol{\beta})\mathbf{T}^{n} + \Delta\hat{\boldsymbol{\Omega}}) \tag{5}$$

The time increment is denoted by Δt. The unsteady calculation can be performed using equation (5).

The performance function for the identification is expressed by the integral of square residuals between calculated and observed temperature as follows:

$$J = \frac{1}{2}\int_{t_0}^{t_f} (\mathbf{T}(\boldsymbol{\beta}) - \mathbf{T}^*)^{\mathrm{T}} (\mathbf{T}(\boldsymbol{\beta}) - \mathbf{T}^*)\,\mathrm{d}t \tag{6}$$

where $\mathbf{T}(\boldsymbol{\beta})$ and $\mathbf{T}^*$ mean calculated and observed temperature, t_0 and t_f denote starting and final time respectively. The parameter $\boldsymbol{\beta}$ that minimizes the performance function J can be obtained by the iterative calculation. Although there are many minimization techniques, the Fletcher–Reeves method that is a sort of the conjugate gradient method is used in this analysis. The search direction $\mathbf{d}$ at the first step is computed by the following equation:

$$\begin{aligned}\mathbf{d}^{(0)} &= -\left\{\frac{\partial J}{\partial \boldsymbol{\beta}^{(0)}}\right\} \\ &= -\int_{t_0}^{t_f}\left[\frac{\partial \mathbf{T}(\boldsymbol{\beta}^{(0)})}{\partial \boldsymbol{\beta}^{(0)}}\right]^{\mathrm{T}} \{\mathbf{T}(\boldsymbol{\beta}^{(0)} - \mathbf{T}^*\}\,\mathrm{d}t\end{aligned} \tag{7}$$

where $[\partial \mathbf{T}(\boldsymbol{\beta})/\partial \boldsymbol{\beta}]$ is referred to as the sensitivity matrix which is obtained by partially differentiating equation (5) with respect to parameter $\boldsymbol{\beta}$ and applying the explicit Euler method, i.e.

$$\left[\frac{\partial \mathbf{T}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right]^{n+1} = \left[\frac{\partial \mathbf{T}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right]^{n} - \Delta t\overline{\mathbf{M}(\gamma)}^{-1}\left(\left[\frac{\partial \mathbf{S}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right]\mathbf{T}^{n}(\boldsymbol{\beta}) + \mathbf{S}(\boldsymbol{\beta})\left[\frac{\partial \mathbf{T}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right]^{n} - \left[\frac{\partial \Delta\hat{\boldsymbol{\Omega}}}{\partial \boldsymbol{\beta}}\right]\right) \tag{8}$$

Initial condition and boundary condition of the sensitive matrix are written as in the following equations:

$$\left[\frac{\partial \mathbf{T}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right]_{t=t_0} = 0 \tag{9}$$

$$\left[\frac{\partial \mathbf{T}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right] = 0 \quad \text{on } \Gamma_1 \tag{10}$$

Consider the performance function $J(\boldsymbol{\beta} + \alpha\mathbf{d})$:

$$\begin{aligned}J(\boldsymbol{\beta} + \alpha\mathbf{d}) &= J + \alpha\left\{\frac{\partial J}{\partial \boldsymbol{\beta}}\right\}^{\mathrm{T}}\mathbf{d} \\ &= \frac{1}{2}\int_{t_0}^{t_f}\left(\mathbf{T}(\boldsymbol{\beta}) + \alpha\left[\frac{\partial \mathbf{T}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right]\mathbf{d} - \mathbf{T}^*\right)^{\mathrm{T}}\left(\mathbf{T}(\boldsymbol{\beta}) + \alpha\left[\frac{\partial \mathbf{T}(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}}\right]\mathbf{d} - \mathbf{T}^*\right)\mathrm{d}t.\end{aligned} \tag{11}$$

The scalar value α can be determined by minimizing $J(\boldsymbol{\beta} + \alpha\mathbf{d})$. Thus, the scalar α is obtained by partially differentiating $J(\boldsymbol{\beta} + \alpha\mathbf{d})$ with respect to α and setting the resulting equation equal to zero:

$$\alpha = -\frac{\mathbf{d}^{\mathrm{T}}\{\partial J/\partial \boldsymbol{\beta}\}}{\mathbf{d}^{\mathrm{T}}\int_{t_0}^{t_f}[\partial \mathbf{T}(\boldsymbol{\beta})/\partial \boldsymbol{\beta}]\,[\partial \mathbf{T}(\boldsymbol{\beta})/\partial \boldsymbol{\beta}]^{\mathrm{T}}\,\mathrm{d}t\,\mathbf{d}} \tag{12}$$

The parameter $\boldsymbol{\beta}$ is renewed using $\mathbf{d}$ and α, which are obtained by equations (7) and (12) respectively. The modified parameter $\boldsymbol{\beta}^{(i+1)}$ is expressed as

$$\boldsymbol{\beta}^{(i+1)} = \boldsymbol{\beta}^{(i)} + \alpha \mathbf{d}^{(i)} \tag{13}$$

The search direction $\mathbf{d}$ is calculated and renewed by the Fletcher–Reeves method, which is expressed as follows:

$$\mathbf{d}^{(i+1)} = -\left\{\frac{\partial J}{\partial \boldsymbol{\beta}}\right\}^{(i+1)} + \varphi\, \mathbf{d}^{(i)} \tag{14}$$

where

$$\varphi = \frac{(\{\partial J/\partial \boldsymbol{\beta}\}^{(i+1)}, \{\partial J/\partial \boldsymbol{\beta}\}^{(i+1)})}{(\{\partial J/\partial \boldsymbol{\beta}\}^{(i)}, \{\partial J/\partial \boldsymbol{\beta}\}^{(i)})} \tag{15}$$

This renewed $\mathbf{d}^{(i+1)}$ is used for the search direction at the next iterative state $i + 1$. The present iterative calculation algorithm is summarized as follows:

1. Assume initial value $\boldsymbol{\beta}^{(0)}$, and calculate $\mathbf{T}(\boldsymbol{\beta}^{(0)})$ and $[\partial \mathbf{T}/\partial \boldsymbol{\beta}^{(0)}]$ by equations (5) and (8).
2. Compute $\mathbf{d}^{(0)} = -\{\partial J/\partial \boldsymbol{\beta}^{(0)}\}$ by equation (7).
3. Determine α that minimizes $J(\boldsymbol{\beta}^{(i)} + \alpha \mathbf{d}^{(i)})$ by equation (12).
4. Compute $\boldsymbol{\beta}^{(i+1)}$ by equation (13).
5. Calculate $\mathbf{T}(\boldsymbol{\beta}^{(i+1)})$ and $[\partial \mathbf{T}(\boldsymbol{\beta}^{(i+1)})/\partial \boldsymbol{\beta}^{(i+1)}]$ by equations (5) and (8).
6. Compute $\varphi = \dfrac{(\{\partial J/\partial \boldsymbol{\beta}\}^{(i+1)}, \{\partial J/\partial \boldsymbol{\beta}\}^{(i+1)})}{(\{\partial J/\partial \boldsymbol{\beta}\}^{(i)}, \{\partial J/\partial \boldsymbol{\beta}\}^{(i)})}$ by equation (15).
7. Compute $\mathbf{d}^{(i+1)}$ by equation (14). If $|\mathbf{d}^{(i+1)}| < \varepsilon$, then stop.
8. Set $i = i + 1$ and go to 3.

OPTIMAL CONTROL

The finite element equation for the optimal control problem is expressed by applying the Galerkin method and implicit Euler scheme to equation (1):

$$\mathbf{T}_{k+1} = (\mathbf{M} + \Delta t \mathbf{S})^{-1} (\mathbf{M}\mathbf{T}_k + \Delta \hat{\boldsymbol{\Omega}}) \tag{16}$$

where $\mathbf{M}$, $\mathbf{S}$, $\Delta \hat{\boldsymbol{\Omega}}$ and Δt denote mass matrix, stiffness matrix, thermal load vector and time increment respectively. The time stage is expressed by k. The state equation is obtained by dividing the finite element equation to the state term, the control term and the force term, i.e.

$$\mathbf{T}_{k+1} = \mathbf{A}\mathbf{T}_k + \mathbf{B}\mathbf{u}_k + \mathbf{C}\mathbf{f}_k \tag{17}$$

where $\mathbf{T}$ is the n-dimensional state vector, $\mathbf{u}$ is the k-dimensional control vector, and $\mathbf{f}$ is the m-dimensional force vector. $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are $n \times n$, $n \times k$ and $n \times m$ matrices.

The performance function for the discrete system is defined by the sum of the square residuals between the calculated and the desirable temperature and of the temperature at the control points:

$$J = \sum_{k=0}^{K} [(\mathbf{T}_k - \mathbf{T}^*)^{\mathrm{T}} \mathbf{Q} (\mathbf{T}_k - \mathbf{T}^*) + (\mathbf{u}_k - \mathbf{u}^*)^{\mathrm{T}} \mathbf{R} (\mathbf{u}_k - \mathbf{u}^*)]$$
$$+ (\mathbf{T}_{K+1} - \mathbf{T}^*)^{\mathrm{T}} Z (\mathbf{T}_{K+1} - \mathbf{T}^*) \tag{18}$$

where $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{Z}$ are symmetric weighting matrices. $\mathbf{T}^*$ and $\mathbf{u}^*$ are objective vectors of state and control values. These values are defined as not random but deterministic ones.

The aim of an optimal control problem is to find a control value $\mathbf{u}^{\mathrm{opt}}$ so as to minimize the performance function. The solution technique to be used in this paper is the discrete-time dynamic programming.[4,5]

Now, define $V(\mathbf{T}, k)$ by

$$V(\mathbf{T}, k) = \min_{\mathbf{u}} \left(\sum_{j=k}^{K} [(\mathbf{T}_j - \mathbf{T}^*)^{\mathrm{T}} \mathbf{Q}(\mathbf{T}_j - \mathbf{T}^*) + (\mathbf{u}_j - \mathbf{u}^*)^{\mathrm{T}} \mathbf{R}(\mathbf{u}_j - \mathbf{u}^*)] + (\mathbf{T}_{K+1} - \mathbf{T}^*)^{\mathrm{T}} Z(\mathbf{T}_{K+1} - \mathbf{T}^*) \right) \tag{19}$$

Equation (19) is rearranged by dividing the term of the kth step and terms after the $(k+1)$th step:

$$\begin{aligned} V(\mathbf{T}, k) &= \min_{\mathbf{u}_k} \Bigg\{ (\mathbf{T}_k - \mathbf{T}^*)^{\mathrm{T}} \mathbf{Q}(\mathbf{T}_k - \mathbf{T}^*) + (\mathbf{u}_k - \mathbf{u}^*) \mathbf{R}(\mathbf{u}_k - \mathbf{u}^*) \\ &\quad + \min_{\mathbf{u}} \left(\sum_{j=k+1}^{K} [(\mathbf{T}_j - \mathbf{T}^*)^{\mathrm{T}} \mathbf{Q}(\mathbf{T}_j - \mathbf{T}^*) + (\mathbf{u}_j - \mathbf{u}^*)^{\mathrm{T}} \mathbf{R}(\mathbf{u}_j - \mathbf{u}^*)] \right. \\ &\quad \left. + (\mathbf{T}_{K+1} - \mathbf{T}^*)^{\mathrm{T}} Z(\mathbf{T}_{K+1} - \mathbf{T}^*) \right) \Bigg\} \\ &= \min_{\mathbf{u}_k} \{ (\mathbf{T}_k - \mathbf{T}^*)^{\mathrm{T}} \mathbf{Q}(\mathbf{T}_k - \mathbf{T}^*) + (\mathbf{u}_k - \mathbf{u}^*)^{\mathrm{T}} \mathbf{R}(\mathbf{u}_k - \mathbf{u}^*) + V(\mathbf{T}_{K+1}, k+1) \} \end{aligned} \tag{20}$$

This is the basic dynamic programming recursive relation.

Equation (20) is solved backwards in time domain starting with the following terminal condition:

$$V(\mathbf{T}, K+1) = (\mathbf{T} - \mathbf{T}^*)^{\mathrm{T}} \mathbf{Z}(\mathbf{T} - \mathbf{T}^*) \tag{21}$$

It is difficult to obtain an optimal control value by solving equation (20) directly. Thus the form of $V(\mathbf{T}, k)$ is assumed as

$$V(\mathbf{T}, k) = b_k + 2\mathbf{p}_k^{\mathrm{T}} \mathbf{T} + \mathbf{T}^{\mathrm{T}} \mathbf{P}_k \mathbf{T} \tag{22}$$

where b_k is a scalar, $\mathbf{p}_k$ is an n-dimensional vector and $\mathbf{P}_k$ is an $n \times n$ symmetric matrix. Because $V(\mathbf{T}, k)$ is quadratic in $\mathbf{T}$, the single stage of the performance function is quadratic in $\mathbf{T}_k$ and $\mathbf{u}_k$, and the state equation (17) is linear in $\mathbf{T}_k$ and $\mathbf{u}_k$, it is reasonable to expect that $V(\mathbf{T}, k)$ will be a quadratic function of $\mathbf{T}$ as in equation (22). Then, the basic recursive relation can be written as

$$\begin{aligned} V(\mathbf{T}, k) = \min_{\mathbf{u}_k} \{ &(\mathbf{T}_k - \mathbf{T}^*)^{\mathrm{T}} \mathbf{Q}(\mathbf{T}_k - \mathbf{T}^*) + (\mathbf{u}_k - \mathbf{u}^*)^{\mathrm{T}} \mathbf{R}(\mathbf{u}_k - \mathbf{u}^*) \\ &+ b_{k+1} + 2\mathbf{p}_{k+1}^{\mathrm{T}} \mathbf{T}_{K+1} + \mathbf{T}_{k+1}^{\mathrm{T}} \mathbf{P}_{k+1} \mathbf{T}_{k+1} \} \end{aligned} \tag{23}$$

Substituting equation (17) into equation (23) yields

$$\begin{aligned} V(\mathbf{T}, k) = \min_{\mathbf{u}_k} \{ &(\mathbf{T}_k - \mathbf{T}^*)^{\mathrm{T}} \mathbf{Q}(\mathbf{T}_k - \mathbf{T}^*) + (\mathbf{u}_k - \mathbf{u}^*)^{\mathrm{T}} \mathbf{R}(\mathbf{u}_k - \mathbf{u}^*) \\ &+ b_{k+1} + 2\mathbf{p}_{k+1}^{\mathrm{T}} (\mathbf{AT} + \mathbf{Bu}_k + \mathbf{Cf}_k) \\ &+ (\mathbf{AT} + \mathbf{Bu}_k + \mathbf{Cf}_k)^{\mathrm{T}} \mathbf{P}_{k+1} (\mathbf{AT} + \mathbf{Bu}_k + \mathbf{Cf}_k) \} \end{aligned} \tag{24}$$

The optimal control value can be obtained by differentiating the right-hand side of equation (24) with respect to $\mathbf{u}_k$ and setting the result equal to zero:

$$\mathbf{u}_k = -(\mathbf{R} + \mathbf{B}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B})^{-1}(\mathbf{B}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{A}\mathbf{T} + \mathbf{B}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{C}\mathbf{f}_k + \mathbf{B}^{\mathrm{T}}\mathbf{p}_{k+1} - \mathbf{R}\mathbf{u}^*) \tag{25}$$

Substituting equations (22) and (25) into equation (24), and rearranging about $\mathbf{P}_k$ and $\mathbf{p}_k$, the Riccati equations are derived and written as follows:

$$\mathbf{P}_k = \mathbf{Q} + \mathbf{A}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{A} - \mathbf{A}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B}(\mathbf{R} + \mathbf{B}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{A} \tag{26}$$

$$\begin{aligned}\mathbf{p}_k = {} & \mathbf{A}^{\mathrm{T}}\mathbf{p}_{k+1} + \mathbf{A}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{C}\mathbf{f}_k - \mathbf{Q}\mathbf{T}^* \\ & - \mathbf{A}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B}(\mathbf{R} + \mathbf{B}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{C}\mathbf{f}_k \\ & - \mathbf{A}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B}(\mathbf{R} + \mathbf{B}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{p}_{k+1} \\ & + \mathbf{A}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B}(\mathbf{R} + \mathbf{B}^{\mathrm{T}}\mathbf{P}_{k+1}\mathbf{B})^{-1}\mathbf{R}\mathbf{u}^*\end{aligned} \tag{27}$$

The terminal conditions for equations (26) and (27) are

$$\mathbf{P}_{K+1} = \mathbf{Z} \tag{28}$$

$$\mathbf{p}_{K+1} = -\mathbf{Z}\mathbf{T}^* \tag{29}$$

Equations (26) and (27) are solved backwards in time domain subjected to these terminal conditions.

NUMERICAL EXAMPLES

To obtain the temperature of the ground, thermometers are buried in the lawn field at the experimental field. The cross-sectional view of the field is shown in Figure 2. The depths of thermometers are shown in Table I.

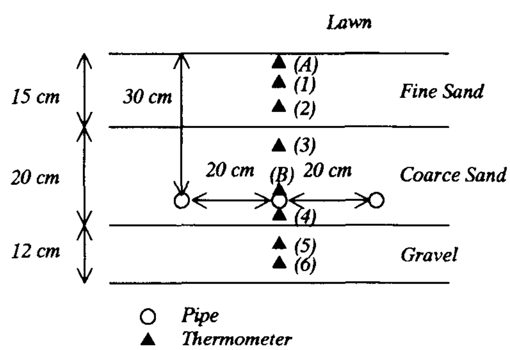

Figure. 2. Cross-sectional view of the field

Table I. Depth of thermometers

	A	1	2	3	B	4	5	6
Depth	2	4	12	20	30	34	40	44

Thermometer A is set at a depth of 2 cm from the ground surface. Thermometer B is stuck to the surface of the pipe of which the depth is 30 cm from the ground surface.

The ground consists of three layers. The first layer is made up of fine sand. The second layer is made up of coarse sand. The third layer is made up of gravel. Thermometers are set more than two in each layer. Temperatures of the field are measured every 30 min.

The finite element mesh for the identification and the control problems is as indicated in Figure 3. In the analysis, it is assumed that the thermal fluxes are zero on the boundary A–B, B–C and C–D. On the A–D boundary and pipe node, observed data at thermometers A and B are specified as the Dirichlet boundary condition. The data observed at the Agricultural Institute of Chiba Prefecture from September 17 to October 7 are used for the calculation in case 1. The data observed at the Institute from December 7 to 24 are used for the calculation in case 2. The time histories of these data are shown in Figures 4 and 5 respectively. It is assumed that the volumetric heat capacity ρC is constant, $2{\cdot}0 \times 10^{6}$ (kg/K m s^2). The identification is performed for the parameter $\boldsymbol{\beta}$ only.

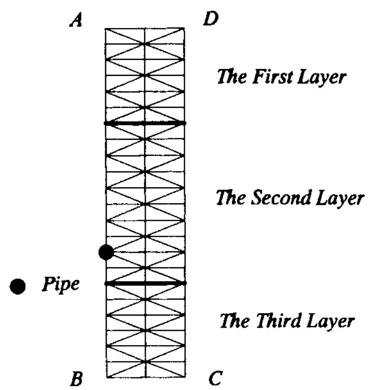

Figure. 3. Finite element mesh

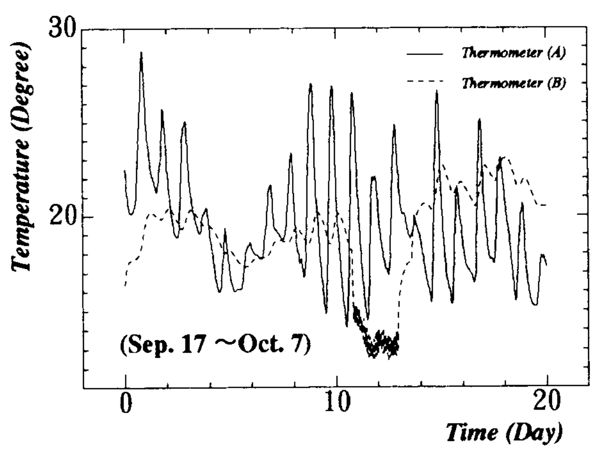

Figure. 4. Time history of observed data A and B

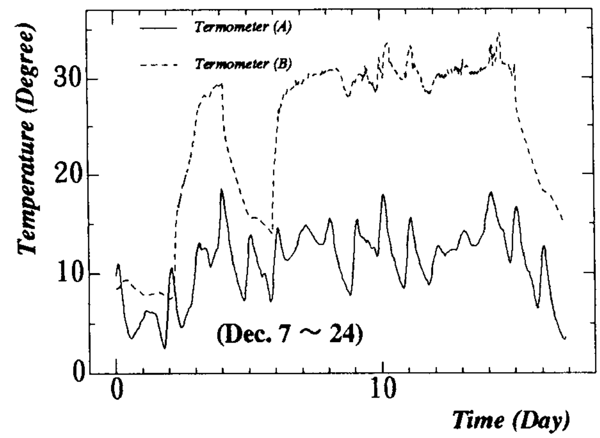

Figure. 5. Time history of thermometers A and B

Computed results of the parameter identification for both data are shown in Figures 6 and 7. From these results, the parameter for each layer can be seen getting good convergence. The circumstances of convergence about the performance function for both cases are indicated in Figures 8 and 9. The converged values of parameters and the performance function are written in Table II.

Figures 10, 11 and 12 show the differences between the computational results using identified parameters and the observed data at the location of thermometers 1, 3 and 5 in case 1, respectively.

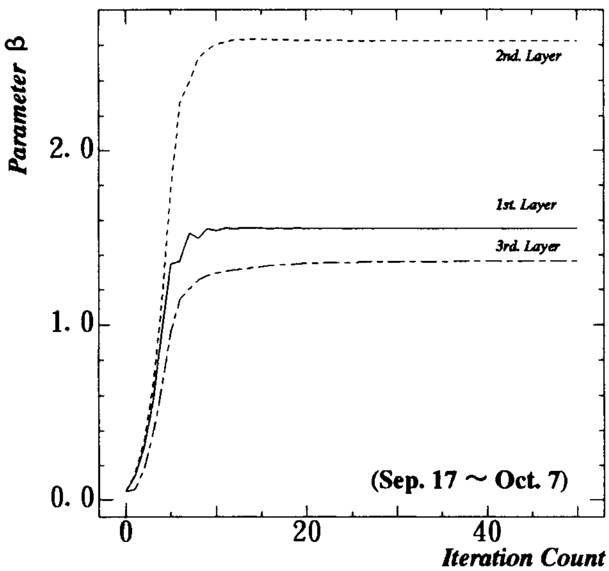

Figure. 6. Calculated parameters

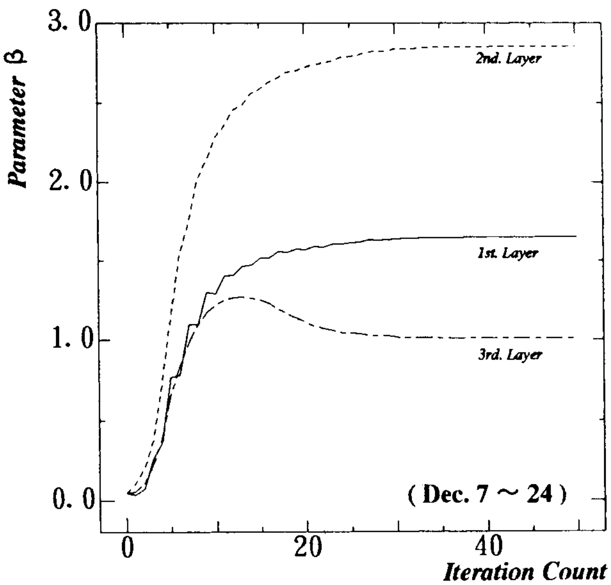

Figure. 7. Calculated parameter

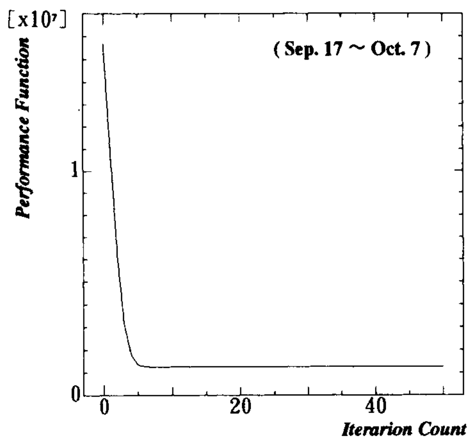

Figure. 8. Convergence of performance function

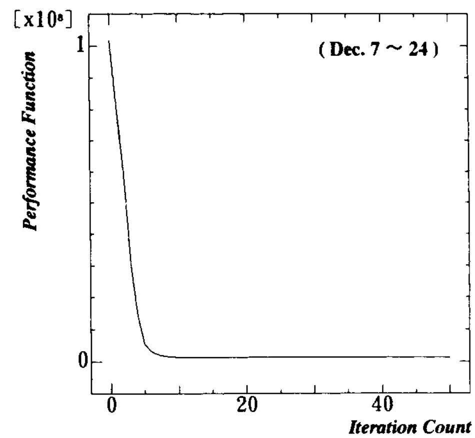

Figure. 9. Convergence of performance function

Table II. Converged parameter and performance function

	Parameter			Performance function	
	β_1	β_2	β_3	Before	After
Case 1	1·55	2·62	1·36	$1{\cdot}57 \times 10^7$	$1{\cdot}27 \times 10^6$
Case 2	1·65	2·85	1·01	$1{\cdot}02 \times 10^8$	$1{\cdot}30 \times 10^6$

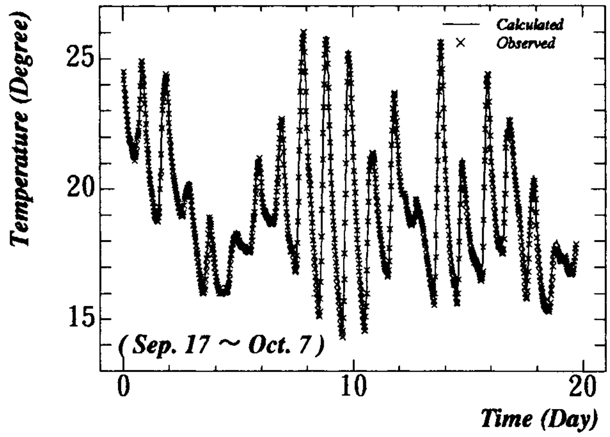

Figure. 10. Difference between calculated and observed temperature at point 1

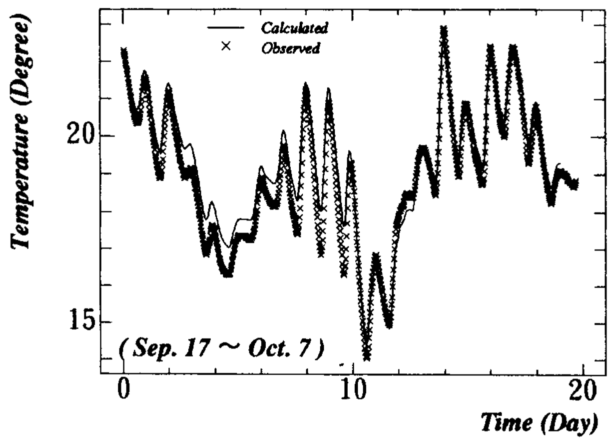

Figure. 11. Difference between calculated and observed temperature at point 3

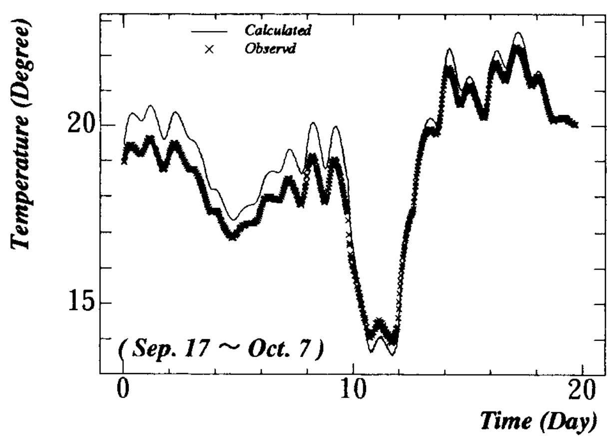

Figure. 12. Difference between calculated and observed temperature at point 5

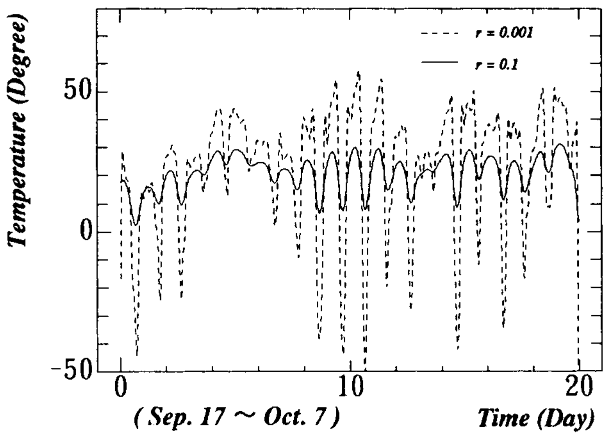

Figure. 13. Optimal control temperature

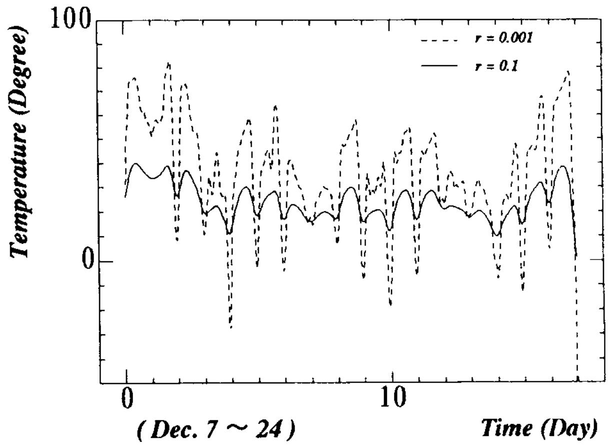

Figure. 14. Optimal control temperature

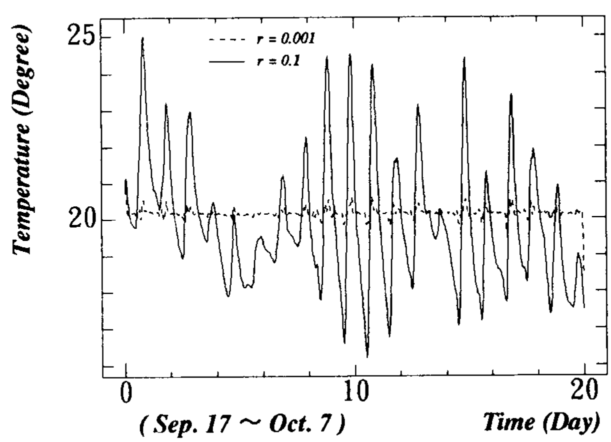

Figure. 15. Temperature at the objective point

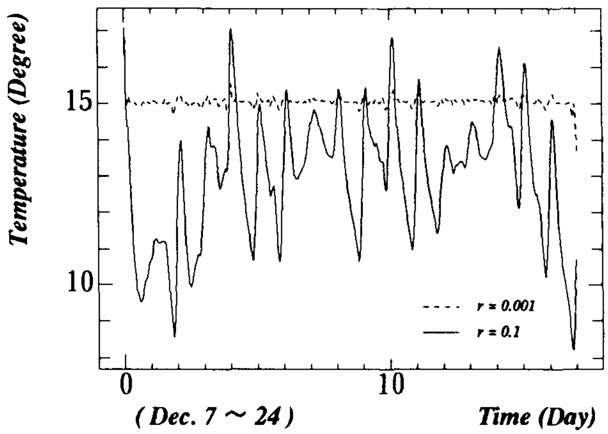

Figure. 16. Temperature at the objective point

In the control problem, diagonal terms which relate the objective points of the weighting matrices **Q** and **Z** are chosen as 1·0. Only the diagonal terms of the **R** matrix are changed to 0·1 or 0·001. Parameter $\boldsymbol{\beta}$ obtained by the parameter identification problem is used.

In case 1, objective values $\mathbf{T}^*$ and $\mathbf{u}^*$ are assumed to be 20° and 15° respectively. In case 2, objective values $\mathbf{T}^*$ and $\mathbf{u}^*$ are assumed to be 15° and 10°. Computed results of the optimal control trajectories are shown in Figures 13 and 14. The time histories of the temperature at the object points whose depth is 6 cm from the ground surface are shown in Figures 15 and 16.

CONCLUDING REMARKS

Using the data for relatively long-term observation, the stable calculations can be carried out. The numerical results show that the obtained parameter based on the observed data in fall is almost the same as that in winter. Comparison between the calculated and the observed temperature shows good agreement at each observation point. However, the parameter β_3 at the third layer is sometimes unstable, because the gradient of temperature at the third layer is insensitive, and the computations may depend on the location of observation points. The thermal conductivity seems independent of the seasonal change of temperature. Further investigation will be the future subject for parameter identification problems.

If the calculation is performed using the identified thermal conductivity, it is possible to predict the exact heat conduction phenomenon in the ground by numerical analysis. The results can be used for the development of the thermal management system.

In the control analysis, in the case that the diagonal term of the **R** matrix, $r = 0{\cdot}001$, is used, the temperature of the objective point is almost the same as the objective temperature $\mathbf{T}^*$ in fall and winter.

The temperature at the objective point can be controlled close to the desired temperature. However. the control temperature should be raised and lowered sharply and suddenly. Also, frequent switching is required. The control temperature is wholly dependent on the capacity of the cooling machine.

In the case that $r = 0{\cdot}1$ is used, the temperature discrepancy between that calculated at the objective point and that given as the objective temperature is greater. On the contrary, the control temperature obtained is within a reasonable practical range, which does not seem impossible using the ordinary cooling machine. The selection of the diagonal terms of the weighting matrix is closely related to the performance of the control system.

It is seen that the construction of the temperature management system can be established based on the method presented in the present paper. The system can be adaptable to other practical problems such as the antifreezing system around a liquid natural gas (LNG) tank.

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